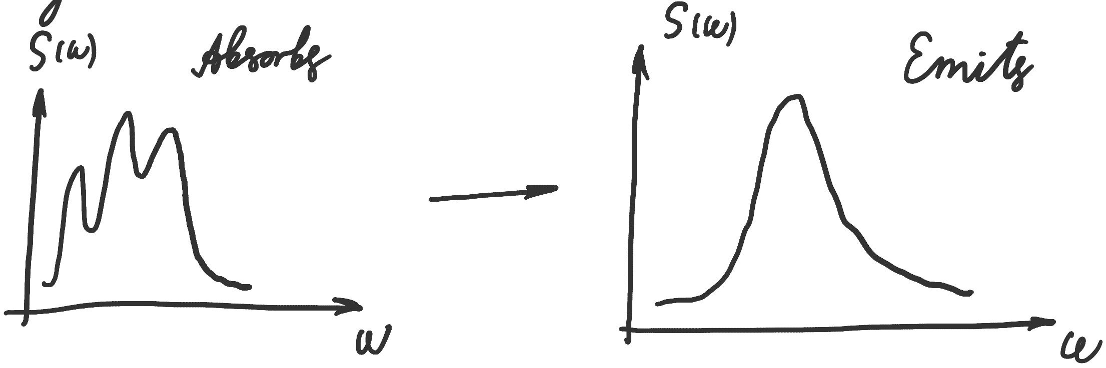
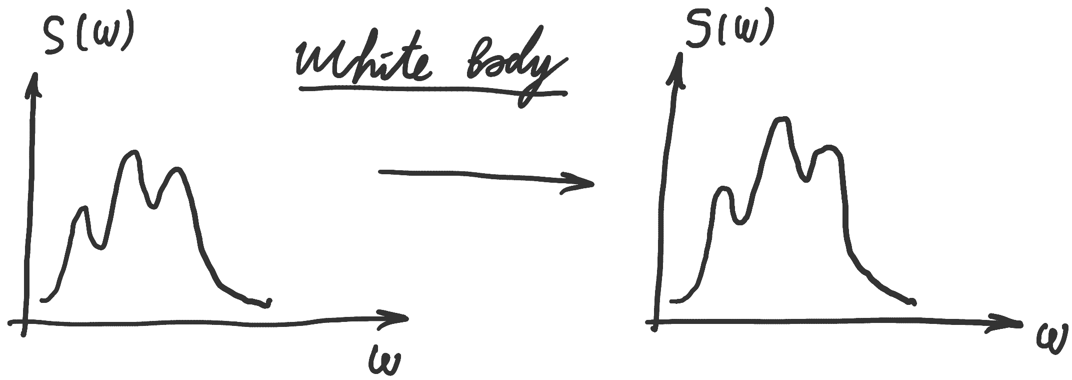
Black-body radiation

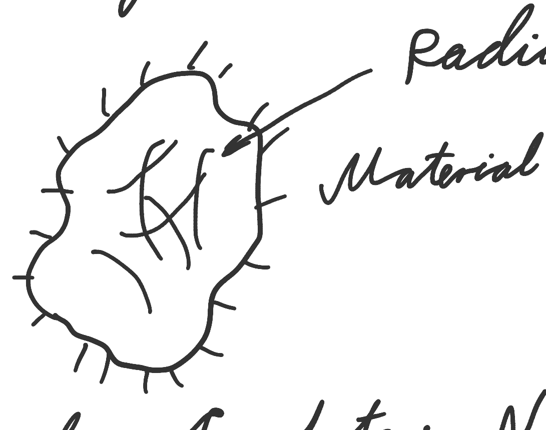
Black body – absorbs all incident radiation, regardless of its spectrum





Examples of black bodies: most stars and planets

The electromagnetic radiation of a black body is a gas of phonons – an ideal gas for all practical purposes



The number of photons N is not conserved

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T and V fixed

F has to be at a minimum

$$\frac{\partial F}{\partial N} = 0 \longrightarrow \mu = 0$$

The energy of a photon is $\varepsilon = \hbar\omega = \hbar c k$

$$\boxed{\bar{n}_k = \frac{1}{e^{\frac{\hbar\omega}{T}} - 1}}$$ – Plank's distribution

The number of momentum states in a spherical layer is

$$\frac{4\pi k^2 dk}{(2\pi)^3} V$$ (Note: k is a wavevector, not momentum)

There is additional degeneracy 2 due to 2 polarisations. So, the number of photons in the wavevector interval dk is

$$dN = \frac{k^2 dk}{\pi^2} V \frac{1}{e^{\frac{\hbar\omega_k}{T}} - 1}$$

$$dN_\omega = \frac{V}{\pi^2 c^3} \frac{\omega^2 d\omega}{e^{\frac{\hbar\omega}{T}} - 1}$$

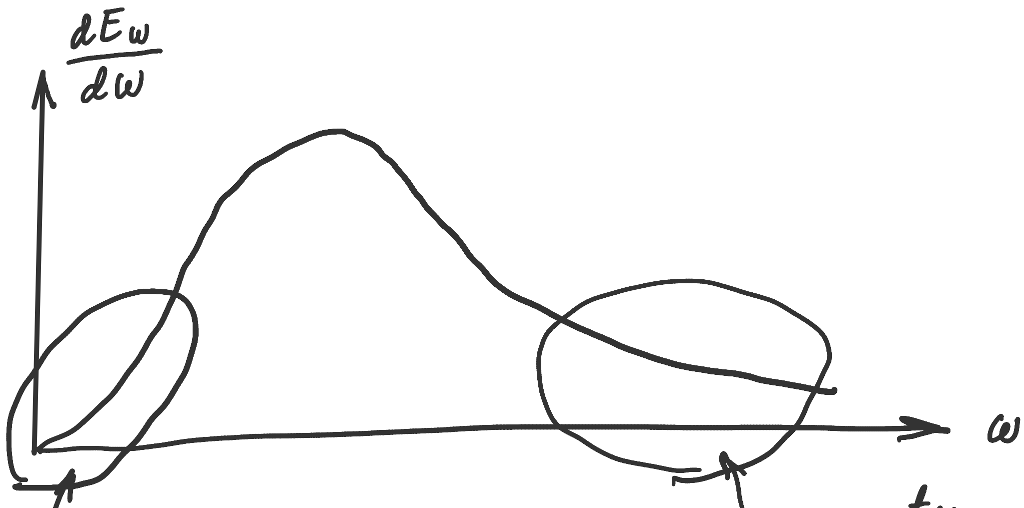
The amount of energy in the interval dω

The amount of energy in the interval $d\omega$ is

$$\boxed{dE_\omega = \frac{V\hbar}{\pi^2 c^3}\frac{\omega^3\, d\omega}{e^{\frac{\hbar\omega}{T}} - 1}}$$ – Planck's law

Express it terms of λ ; $\omega = \frac{2\pi c}{\lambda}$

$$\boxed{dE_\lambda = \frac{16\pi^2 c\hbar V}{\lambda^5}\frac{d\lambda}{e^{\frac{2\pi\hbar c}{T\lambda}} - 1}}$$



$\frac{VT}{\pi^2 c^3}\omega^2$

$\frac{V\hbar}{\pi^2 c^3}\omega^3 e^{-\frac{\hbar\omega}{T}}$

Wien's formula

// There are $\frac{T}{\omega}$ photons = vibrations of the media

Rayleigh-Jeans formula

$\hbar\omega$

Rayleigh - Jeans

$$F = \Omega = T \frac{V}{\pi^2 c^3} \int_0^\infty \omega^2 \ln\left(1 - e^{-\frac{\hbar\omega}{T}}\right) d\omega =$$

$$= \frac{V T^4}{\pi^2 c^3 \hbar^3} \int_0^\infty z^2 \ln(1 - e^{-z}) dz =$$

$z = \frac{\hbar\omega}{T}$

$$= \frac{V T^4}{\pi^2 c^3 \hbar^3} \frac{z^3}{3} \ln(1 - e^{-z}) \Big|_0^\infty - \frac{V T^4}{3\pi^2 c^3 \hbar^3} \underbrace{\int \frac{z^3}{e^z - 1} dz}_{\frac{\pi^4}{15}}$$

$$F = -V \frac{\pi^2 T^4}{45 (\hbar c)^3} = -V \cdot \frac{4\sigma T^4}{3c}$$

σ - Stefan-Boltzmann constant

$$S = -\frac{\partial F}{\partial T} = V \cdot \frac{16\sigma T^3}{3c}$$

$$E = F + TS = V \frac{4\sigma T^4}{c} \qquad (= -3F)$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{16\sigma T^3 V}{c}$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{4\sigma T^4}{3c}$$

$$PV = \frac{E}{3}$$

Adiabat equation : $VT^3 = const$